8.3 Relative Rates of Growth

Definitions: Faster, Slower, Same-rate Growth

if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \infty$ then f(x) faster

if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$ then f(x) slower

if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = c$ then same

$0 < c < \infty$

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Which function grows faster?

e^x, x^2 $\lim_{x\to\infty} \frac{e^x}{x^2} = \frac{\infty}{\infty}$ indet

$\lim_{x\to\infty} \frac{e^x}{2x} = \lim_{x\to\infty} \frac{e^x}{2} = \infty$

e^x grows faster than x^2

$\ln x, x, x^2$

$\lim_{x\to\infty} \frac{\ln x}{x} = \lim_{x\to\infty} \frac{1/x}{1} = 0$

so $\ln x$ slower than x (growing)

$\lim_{x\to\infty} \frac{x}{x^2} = \lim_{x\to\infty} \frac{1}{x} = 0$

so x grows slower than x^2

x is a lower order of magnitude than x^2

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Show the functions grow at the same rate

$x, x + \sin x$

$$\lim_{x\to\infty} \frac{x+\sin x}{x} = \lim_{x\to\infty} \frac{1+\cos x}{1} = ? \le 2$$

$$\lim_{x\to\infty} \frac{x}{x} + \frac{\sin x}{x} = 1+0 = 1$$

$\log_a x, \log_b x$

$$\lim_{x\to\infty} \frac{\log_a x}{\log_b x} = \lim_{x\to\infty} \frac{\frac{1}{x\ln a}}{\frac{1}{x\ln b}}$$

$\frac{\infty}{\infty}$ Indet.

$$= \lim_{x\to\infty} \frac{1}{x\ln a} \cdot \frac{x\ln b}{1}$$

same rate

$$= \frac{\ln b}{\ln a}$$

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Transitivity of Growing Rates

If $a=b$ and $b=c$ then $a=c$

Show the functions grow at the same rate by comparing both with x

$\sqrt{x^2+5}, (2\sqrt{x}-1)^2$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+5}}{\sqrt{x^2}} = \lim_{x\to\infty} \sqrt{\frac{x^2+5}{x^2}} = \sqrt{1} = 1$$

same

1

$$\lim_{x\to\infty} \frac{(2\sqrt{x}-1)^2}{(\sqrt{x})^2} = \lim_{x\to\infty} \left(\frac{2\sqrt{x}-1}{\sqrt{x}}\right)^2 = 2^2$$

2

same

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Sequential vs Binary Searches

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